## GUJARAT TECHNOLOGICAL UNIVERSITY

AUTOMOBILE ENGINEERING (02), INDUSTRIAL ENGINEERING (15) \& MECHANICAL ENGINEERING (19)<br>COMPLEX VARIABLES AND NUMERICAL METHODS<br>SUBJECT CODE: 2141905<br>B.E. $4^{\text {th }}$ SEMESTER

Type of course: Engineering Mathematics
Prerequisite: As a pre-requisite to this course students are required to have a reasonable mastery over multivariable calculus, differential equations and Linear algebra

## Rationale:

Mathematics is a language of Science and Engineering.
Teaching and Examination Scheme:

| Teaching Scheme |  |  | Credits | Examination Marks |  |  |  |  |  | Total Marks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| L | T | P | C | Theory Marks |  |  | Practical Marks |  |  |  |
|  |  |  |  |  |  |  |  |  | PA |  |
|  |  |  |  | (E) | PA | ALA | ESE | OEP | (I) |  |
| 3 | 2 | 0 | 5 | 70 | 20 | 10 | 30 | 0 | 20 | 150 |

## Content:

| Sr. No. | Content | $\begin{gathered} \hline \text { Total } \\ \text { Hrs } \\ \hline \end{gathered}$ | \% <br> Weightage |
| :---: | :---: | :---: | :---: |
| 1 | Complex Numbers and Functions: <br> Exponential, Trigonometric, De Moivre's Theorem, Roots of a complex number ,Hyperbolic functions and their properties, Multi-valued function and its branches: Logarithmic function and Complex Exponent function Limit, Continuity and Differentiability of complex function, Analytic functions, Cauchy-Riemann Equations, Necessary and Sufficient condition for analyticity, Properties of Analytic functions, Laplace Equation, Harmonic Functions, Harmonic Conjugate functions and their Engineering Applications | 10 | 24 |
| 2 | Complex Integration: <br> Curves, Line Integral(contour integral) and its properties, CauchyGoursat Theorem, Cauchy Integral Formula, Liouville Theorem (without proof), Maximum Modulus Theorems(without proof) | 04 | 10 |
| 3 | Power Series: <br> Convergence(Ordinary, Uniform, Absolute) of power series, Taylor and Laurent Theorems (without proof), Laurent series expansions, zeros of analytic functions , Singularities of analytic functions and their classification <br> Residues: Residue Theorem, Rouche's Theorem (without proof) | 05 | 12 |
| 4 | Applications of Contour Integration: <br> Evaluation of various types of definite real integrals using contour | 02 | 5 |


|  | integration method |  |  |
| :---: | :--- | :--- | :--- |
| $\mathbf{5}$ | Conformal Mapping and its Applications: <br>  <br> Magnification, Inversion, Mobius(Bilinear), <br> Schwarz-Christoffel transformations | 03 | 7 |
| $\mathbf{6}$ | Interpolation: Finite Differences, Forward, Backward and Central <br> operators, <br> Interpolation by polynomials: Newton's forward ,Backward interpolation <br> formulae, Newton's divided Gauss \& Stirling's central difference <br> formulae and Lagrange's interpolation formulae for unequal intervals | 04 | 10 |
| $\mathbf{7}$ | Numerical Integration: <br> Newton-Cotes formula, Trapezoidal and Simpson's formulae, error <br> formulae, Gaussian quadrature formulae | 03 | 7 |
| $\mathbf{8}$ | Solution of a System of Linear Equations: Gauss elimination, partial <br> pivoting, Gauss-Jacobi method and Gauss-Seidel method | 03 | 7 |
| $\mathbf{9}$ | Roots of Algebraic and Transcendental Equations: <br> Bisection, false position, Secant and Newton-Raphson <br> methods, Rate of convergence | 03 | 7 |
| $\mathbf{1 0}$ | Eigen values by Power and Jacobi methods | 02 | 4 |
| $\mathbf{1 1}$ | Numerical solution of Ordinary Differential Equations: <br> Euler and Runge-Kutta methods | 7 |  |

## Suggested Specification table with Marks (Theory):

| Distribution of Theory Marks |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| R Level | U Level | A Level | N Level | E Level |
| $10 \%$ | $15 \%$ | $20 \%$ | $20 \%$ | $35 \%$ |

Legends: R: Remembrance; U: Understanding; A: Application, N: Analyze and E: Evaluate and above Levels (Revised Bloom's Taxonomy)

Note: This specification table shall be treated as a general guideline for students and teachers. The actual distribution of marks in the question paper may vary slightly from above table

## Reference Books:

1. R. V. Churchill and J. W. Brown, Complex Variables and Applications (7th Edition), McGraw-Hill (2003)
2. J. M. Howie, Complex Analysis, Springer-Verlag(2004)
3. M. J. Ablowitz and A.S. Fokas, Complex Variables-Introduction and Applications, Cambridge University Press, 1998 (Indian Edition)
4. E. Kreyszig, Advanced Engineering Mathematics(8th Edition), John Wiley (1999)
5. S. D. Conte and Carl de Boor, Elementary Numerical Analysis-An Algorithmic Approach (3rd Edition), McGraw-Hill, 1980
6. C.E. Froberg, Introduction to Numerical Analysis (2nd Edition), Addison-Wesley, 1981
7. Gerald C. F. and Wheatley,P.O., Applied Numerical Analysis (Fifth Edition), Addison-Wesley, Singapore, 1998.
8. Chapra S.C, Canale, R P, Numerical Methods for Engineers , Tata McGraw Hill, 2003

## Course Outcome:

After learning the course the students should be able to:

- evaluate exponential, trigonometric and hyperbolic functions of a complex number
- define continuity, differentiability, analyticity of a function using limits. Determine where a function is continuous/discontinuous, differentiable/non-differentiable, analytic/not analytic or entire/not entire.
- determine whether a real-valued function is harmonic or not. Find the harmonic conjugate of a harmonic function.
- understand the properties of Analytic function.
- evaluate a contour integral with an integrand which have singularities lying inside or outside the simple closed contour.
- recognize and apply the Cauchy's integral formula and the generalized Cauchy's integral formula.
- classify zeros and singularities of an analytic function.
- find the Laurent series of a rational function.
- write a trigonometric integral over $[0,2 \pi]$ as a contour integral and evaluate using the residue theorem.
- distinguish between conformal and non conformal mappings.
- find fixed and critical point of Bilinear Transformation.
- calculate Finite Differences of tabulated data.
- find an approximate solution of algebraic equations using appropriate method.
- find an eigen value using appropriate iterative method.
- find an approximate solution of Ordinary Differential Equations using appropriate iterative method.


## List of Open Source Software/learning website:

http://ocw.mit.edu/resources/res-18-008-calculus-revisited-complex-variables-differential-equations-and-linear-algebra-fall-2011/part-i/
http://nptel.ac.in/courses/111105038/
http://nptel.ac.in/courses/111104030/
http://nptel.ac.in/courses/111107063/
http://nptel.ac.in/courses/111101003/
ACTIVE LEARNING ASSIGNMENTS: Preparation of power-point slides, which include videos, animations, pictures, graphics for better understanding theory and practical work - The faculty will allocate chapters/ parts of chapters to groups of students so that the entire syllabus to be covered. The power-point slides should be put up on the web-site of the College/ Institute, along with the names of the students of the group, the name of the faculty, Department and College on the first slide. The best three works should submit to GTU.

